Conditionals and Closure

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We investigate a tension between *closure*, a compelling norm of rational belief, and plausible assumptions about the semantics of indicative conditionals.

The Principles

The following principle seems compelling:

> CLOSURE. If S is justified to believe that P, P logically entails Q and S competently deduces Q from P, then S is justified to believe Q.

There seems to be an important connection between belief and probability:[1]

> J1. S is justified to believe that P iff the probability of P is very high.

> J2. S is justified to believe *if P, Q* iff the probability of Q given P is very high.

The following inferences seem valid for the indicative conditional:

(1) **Or-to-If**
P or Q.
Therefore, if not-P, then Q.

(2) **Contraposition**
If P, then Q.
Therefore, if not-Q, then not-P.

The Puzzle

Taken together, CLOSURE, J1, J2 and any of the validities above are inconsistent. Consider:

> *Fair coin.* Ann has a fair coin and is about to flip it one million times.

By J1, Ann is justified to believe:

(3) The coin will not land heads exactly once.

Now note that (3) is equivalent to:

(4) Either the coin will land heads at least twice, or the coin will always land tails.

Further, by *Or-to-If* (4) entails

(5) If the coin doesn't land heads at least twice, it will always lands tails.

Suppose that Ann competently deduces (5) from (3) and comes to believe (5) as a result. However, by J2 Ann is *not* justified to believe (5). This is because the conditional probability that the coin always lands tails, given that it doesn't land heads at least twice, is very small. Therefore, CLOSURE fails. (Similar cases can be constructed for Contraposition.)

The Upshot

We face an important choice point: Either we give up CLOSURE, or we give up the validity of these inferences. We argue that we should give up CLOSURE and keep the validity of these inferences, and give an information state semantics which explains why CLOSURE fails.

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[1]Note that in these principles, P and Q are free of epistemic modals and indicative conditionals.